# Reasoning about attribute value equivalence in relational data 

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## A R T I C L E I N F O

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#### Abstract

In relational data, identifying the distinct attribute values that refer to the same real-world entities is an essential task for many data cleaning and mining applications (e.g., duplicate record detection and functional dependency mining). The state-of-the-art approaches for attribute value matching are mainly based on string similarity among attribute values. However, these approaches may not perform well in the cases where the specified string similarity metric is not a reliable indicator for attribute value equivalence. To alleviate such limitations, we propose a new framework for attribute value matching in relational data. Firstly, we propose a novel probabilistic approach to reason about attribute value equivalence by value correlation analysis. We also propose effective methods for probabilistic equivalence reasoning with multiple attributes. Next, we present a unified framework, which incorporates both string similarity measurement and value correlation analysis by evidential reasoning. Finally, we demonstrate the effectiveness of our framework empirically on real-world datasets. Through extensive experiments, we show that our framework outperforms the string-based approaches by considerable margins on matching accuracy and achieves the desired efficiency.


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## 1. Introduction

The same real-world entities may have different representations within or across relational databases. Variations in representation can arise from differences in storage formats, typographical errors, aliases and abbreviations. Determining attribute value equivalence is an essential task for many relational data cleaning and mining applications [1,2]. For instance, most techniques for duplicate record detection in relational data [3,4] divide each record into fields (attributes) and identify duplicate records by comparing their values on fields. Effective attribute value matching can therefore improve the accuracy of duplicate record identification. Functional dependency and conditional functional dependency mining $[5,6]$ also requires attribute value matching to reduce noise: nonidentical but equivalent attribute values could make a valid functional dependency elusive.

As pointed out by the surveys [7,8], most existing work on attribute value matching focused on reasoning about the equivalence between string data. The state-of-the-art techniques are based on measuring string similarity. A wide variety of metrics [9-12] have been proposed for this purpose. In comparison, the methods for capturing similarity in numeric data are rather primitive. Typically,

[^0]the similar numbers are located by simple range queries, or treated as strings, which are then compared using string similarity metrics. Therefore, effective matching usually requires a metric to accommodate the value representation variations specific to a domain. Even though the existing string similarity metrics have been shown to be effective in various applications, they also have the fundamental limitation: a metric tuned and tested on previous problems can perform poorly on a new problem. Even though researchers have proposed adaptive algorithms [ 13,14$]$ that can learn similarity metrics automatically, the difficulty of using these methods cannot be overlooked: they require significant training data and intensive human intervention. Provided with a new problem, it remains challenging to design both string similarity metric and threshold that can effectively capture the value representation variations present in the problem.

We illustrate the limitation of the string-based approach by the example as shown in Table 1. The relational records refer to research papers and each paper has four attributes, title, author, journal, year, which describe the title, authors, publication venue and publication year of the paper respectively. It can be observed that the journal values "Computers" and "Computer" look very similar but actually represent different publication venues. In contrast, the journal values "Journal on Very Large Data Bases" and "VLDB J" appear much less similar but actually refer to the same research journal. To alleviate the limitation of the stringbased approach, we propose to reason about attribute value equiv-
Table 1
A relational table on research publications.

| title | author | yournal |
| :--- | :--- | :--- |
| Energy management in industrial plants | D. Bruneo, A. Cucinotta, A.L. Minnolo, A. Puliafito, M. Scarpa | Computers |
| Beyond bits: the future of quantum information processing | A.M. Steane, E.G. Rieffel | Computer |
| Priority assignment in real-time active databases | R.M. Sivasankaran, J.A. Stankovic, D. Towsley, B. Purimetla, K. Ramamritham | Journal on Very Large Data Bases |
| A taxonomy of correctness criteria in database applications | K. Ramamritham, P.K. Chrysanthis | 20003 |

alence by value correlation analysis. Note that in Table 1, the two title values, which are correlated with "Journal on Very Large Data Bases" and "VLDB J" respectively, have a common keyword "database", and similarly, the author values correlated with them share a common author "K. Ramamritham". Generally, we observe that the papers published in the same journal have a higher probability to be in the same research area than those published in different journals. Accordingly, they usually share some author names and their titles share some common keywords with higher probabilities. As a result, correlation analysis between the journal values and their corresponding author and title values can provide with useful clues for equivalence reasoning. More specifically, if two journal values are correlated with many common author values and many highly similar title values, it can be reasoned that they refer to the same journal entity with a high probability.

Note that a simple type of correlation among attribute values can be described by functional dependency, which specifies that the value of one attribute uniquely determines the value at another attribute. Obviously, a functional dependency can be exploited to match two attribute values. In the example shown in Table 1, suppose that each paper has a unique title. Accordingly, we have the functional dependency,
$\mathrm{fd}_{1}$ : title $\rightarrow$ journal

As a result, two attribute values on journal can be determined to be equivalent if their corresponding records have the same value at the attribute title. Unfortunately, in practice, it is challenging to detect a clear-cut functional dependency in the presence of nonidentical but equivalent attribute values, and even if it can be successfully detected, it may be of limited use in determining equivalence due to lack of matching data. Again, in the example shown in Table 1, if each record refers to a unique paper, the functional dependency, $\mathrm{fd}_{1}$, is then powerless in determining attribute value equivalence at journal because there do not exist two papers sharing a common title.

As illustrated by the motivating example, besides string similarity measurement, value correlation analysis can also be useful in reasoning about attribute value equivalence in relational data. Therefore, in this paper, we aim for a formal probabilistic model for value correlation analysis, and also a unified framework that can incorporate both string similarity measurement and value correlation analysis. Our major contributions can be summarized as follows:

1. We present a novel probabilistic approach to estimate the probability of attribute value equivalence by value correlation analysis, which reasons about the equivalence between two attribute values by analysing their correlation with other attribute values.
2. We propose a unified framework for attribute value matching in relational data. Based on both string similarity measurement and value correlation analysis, it provides a unified equivalence estimation by evidential reasoning. The proposed framework is a unified one in the sense that it can be simplified into a pure string similarity metric by setting the evidence weight of value correlation analysis to be 0 .
3. We experimentally evaluate the performance of the proposed framework on real-world publicly-available datasets. Our extensive experiments show the effectiveness of the unified framework, demonstrating that it outperforms the string-based approaches by considerable margins on matching accuracy and achieves the desired efficiency.

Table 2
Summary of symbols.

| Denotation | Description |
| :--- | :--- |
| $R$ | A relational table. |
| $r \in R$ | A tuple from $R$. |
| $X, Y$ | Correlated and target attribute in $R$. |
| $\mathbf{X}, \mathbf{Y}$ | Domains of attribute $X$ and $Y$ in $R$. |
| $X[R]$ | Attribute values, $\{r[X] \mid r \in R\}$, at $X$ in $R$. |
| $Y[R]$ | Attribute values, $\{r[Y] \mid r \in R\}$, at $Y$ in $R$. |
| $x_{i} \in \mathbf{X}\left(y_{i} \in \mathbf{Y}\right)$ | Values at attribute $X(Y)$. |
| $R\left[x_{i}\right]$ | Tuples $\left\{r \mid r[X]=x_{i}\right\}$ in $R$ who have values $x_{i}$ at attribute $X$. |
| $X\left[y_{j}\right]$ | Attribute values $\left\{r[X] \mid r[Y]=y_{j}\right\}$, at $X$ in $R\left[y_{j}\right]$. |
| $Y\left[x_{i}\right]$ | Attribute values $\left\{r[Y] \mid r[X]=x_{i}\right\}$, at $Y$ in $R\left[x_{i}\right]$. |

## 2. Problem statement

For simplicity, those notations and corresponding meanings used in this paper are shown in Table 2.

Note that in $R$, each non-null value $y_{i}$ at target attribute $Y$ refers to a real-world entities. Two attribute values are deemed to be equivalent, denoted by $y_{i} \simeq y_{j}$, if and only if they refer to the same real-world entity. For instance, in Table 1, the values "VLDB J" and "Journal on Very Large Data Bases" are equivalent because they refer to the same publication venue.

The problem of attribute value matching in relational data is to find the equivalences existing between values at target attribute. It can be formally defined as follows:

Definition 1. Given two domains of distinct relations, $\mathbf{Y}_{1} \in R_{1}$ and $\mathbf{Y}_{2} \in R_{2}$ and pairs of attribute values, $\left\langle y_{i}, y_{j}\right\rangle \in \mathbf{Y}_{1} \times \mathbf{Y}_{2}$, the problem of attribute value matching is to identify all the pairs $\left\langle y_{i}, y_{j}\right\rangle$ that referring to the same real-world entity.

Above definition has already subsumed the special case of finding pairs of equivalent attribute values within a single relation $\left(Y_{1}=Y_{2} \wedge R_{1}=R_{2}\right)$.

Attribute value matching is usually performed by pairwise comparisons. It ranks the pairs of attribute values by their equivalence probabilities. Therefore, the core challenge is to compute the equivalence probability between two given values at target attribute.

## 3. Proposed framework

In this section, we present the concept of Value Correlation Analysis (VCA), and then introduce the unified framework to incorporate the String Similarity Measurement (SSM) and Value Correlation Analysis (VCA).

### 3.1. Value correlation Analysis

We first present a basic model in Section 3.1.1 which exploits only identical values and reasons about value equivalence on a target attribute based on another attribute. Next, in Section 3.1.2, we propose an extended model that incorporates string similarity metrics into the estimation of equivalence reasoning. Finally, in Section 3.1.3, we discuss how to reason about value equivalence on a target attribute by multiple correlated attributes.

We first present a basic model in Section 3.1.1 which exploits only identical values and reasons about value equivalence on a target attribute based on another attribute. Next, in Section 3.1.2, we propose an extended model that incorporates string similarity metrics into the process of equivalence reasoning. Finally, in Section 3.1.3, we discuss how to reason about value equivalence on a target attribute based on multiple attributes.

### 3.1.1. Basic model

Given two non-identical values $y_{1}$ and $y_{2}$ at the target attribute $Y$, the basic model uses their correlated values at another attribute $X$ to estimate their equivalence probability. As shown in Table 2, $X\left[y_{i}\right]$ denotes the attribute values at $X$ for the tuples in $R\left[y_{i}\right]$. Therefore, the attribute values (e.g. $X\left[y_{i}\right]$ ) may contain duplicate attribute values.

Based on the Bayes' theorem, the probability of $y_{1}$ and $y_{2}$ being equivalent can be computed by
$P\left(y_{1} \simeq y_{2}\right)=\frac{P\left(y_{1} \simeq y_{2} \mid x_{1} \simeq x_{2}\right)}{P\left(x_{1} \simeq x_{2} \mid y_{1} \simeq y_{2}\right)} \cdot P\left(x_{1} \simeq x_{2}\right)$
in which $y_{1}, y_{2} \in \mathbf{Y}, P\left(x_{1} \simeq x_{2}\right)$ represents the probability that a value in $X\left[y_{1}\right]$ is equivalent to a value in $X\left[y_{2}\right]$, and $P\left(y_{1} \simeq y_{2} \mid x_{1} \simeq x_{2}\right)$ and $P\left(x_{1} \simeq x_{2} \mid y_{1} \simeq y_{2}\right)$ are two conditional probabilities representing the correlation between two attributes $X$ and $Y$ with respect to $y_{1}$ and $y_{2}$, respectively.

For reasoning about value equivalence between $y_{1}$ and $y_{2}$, the two conditional probabilities at the right-hand side of Eq. (2) cannot be computed directly from data. Therefore, we approximate them by general correlation analysis, which considers all the attribute values at $Y$ and $X$ existing in $R$, as follows:
$\frac{P\left(y_{1} \simeq y_{2} \mid x_{1} \simeq x_{2}\right)}{P\left(x_{1} \simeq x_{2} \mid y_{1} \simeq y_{2}\right)} \approx \frac{P\left(y_{i} \simeq y_{j} \mid x_{i} \simeq x_{j}\right)}{P\left(x_{i} \simeq x_{j} \mid y_{i} \simeq y_{j}\right)}$
in which $P\left(y_{i} \simeq y_{j} \mid x_{i} \simeq x_{j}\right)$ represents the probability that two random tuples' attribute values at $Y$ are equivalent given that their attribute values at $X$ are equivalent, and analogously $P\left(x_{i} \simeq x_{j} \mid y_{i} \simeq y_{j}\right)$ represents the probability that two tuples' attribute values at $X$ are equivalent given that their attribute values at $Y$ are equivalent.

Note that Eq. (3) essentially estimates the conditional probabilities between the individual values, $\left\{y_{1}, y_{2}\right\}$ and $\left\{x_{1}, x_{2}\right\}$, by the collective conditional probability relationship existing between $Y$ and $X$ in $R$. It can be observed that while the left-hand side of Eq. (3) can not be computed directly, the right-hand side can usually be easily computed in practice. For instance, in the running example shown in Table 1, suppose that $Y$ and $X$ correspond to the journal and title attributes respectively, and two attribute values are equivalent if and only if they are identical. Then, $P\left(y_{i} \simeq y_{j} \mid x_{i} \simeq x_{j}\right)$ corresponds to the probability that two paper records have the same journal value given that they have the same title. Similarly, $P\left(x_{i} \simeq x_{j} \mid y_{i} \simeq y_{j}\right)$ corresponds to the probability that two paper records have the same title given that they have the same journal value.

Denoting the right-hand side of Eq. (3) by the general correlation factor between $Y$ and $X, f(Y, X)$, we have
$f(Y, X)=\frac{P\left(y_{i} \simeq y_{j} \mid x_{i} \simeq x_{j}\right)}{P\left(x_{i} \simeq x_{j} \mid y_{i} \simeq y_{j}\right)}$
Accordingly, the equivalence probability estimation for pair $\left\langle y_{1}\right.$, $\left.y_{2}\right\rangle$ can be represented by
$P\left(y_{1} \simeq y_{2}\right) \approx f(Y, X) \cdot P\left(x_{1} \simeq x_{2}\right)$
Note that in Eq. (4), the two conditional probabilities can be represented by
$\begin{cases}P\left(y_{i} \simeq y_{j} \mid x_{i} \simeq x_{j}\right) & =\frac{P\left(y_{i} \sim y_{j} \wedge x_{i} \simeq x_{j}\right)}{\left(P i_{i} \simeq j_{j}\right)} \\ P\left(x_{i} \simeq x_{j} \mid y_{i} \simeq y_{j}\right) & =\frac{P\left(y_{i} \simeq y_{j} \wedge x_{i} \simeq j_{j}\right)}{P\left(y_{i} \simeq y_{j}\right)}\end{cases}$
Therefore, by combining Eq. (4) with Eq. (6), we can represent the correlation factor between $Y$ and $X, f(Y, X)$, as
$f(Y, X)=\frac{P\left(y_{i} \simeq y_{j}\right)}{P\left(x_{i} \simeq x_{j}\right)}$
where $P\left(y_{i} \simeq y_{j}\right)$ represents the equivalence probability of two random values in $Y(R)$, and $P\left(x_{i} \simeq x_{j}\right)$ represents the equivalence probability of two random values in $X(R)$.

In the basic model, we assume that two attribute values are equivalent iff they are identical. It estimates $P\left(x_{1} \simeq x_{2}\right)$ in Eq. (5) by $P\left(x_{1}=x_{2}\right)$. Similarly, it estimates the correlation factor, $f(Y, X)$, by
$f(Y, X)=\frac{P\left(y_{i}=y_{j}\right)}{P\left(x_{i}=x_{j}\right)}$
in which $P\left(y_{i}=y_{j}\right)$ represents the probability of two random values in $Y(R)$ being identical, and $P\left(x_{i}=x_{j}\right)$ represents the probability of two random values in $X(R)$ being identical. Therefore, the equivalence probability estimation of $y_{1}$ and $y_{2}$ can be represented by
$P\left(y_{1} \simeq y_{2}\right) \approx f(Y, X) \cdot P\left(x_{1}=x_{2}\right)$
in which $P\left(x_{1}=x_{2}\right)$ represents the probability of a random value in $X\left[y_{1}\right]$ being identical to a random value in $X\left[y_{2}\right]$, and the value of $f(Y, X)$ is estimated by Eq. (8).

We observe that the computation of $f(Y, X)$ as shown in Eq. (8) requires to retrieve all the $Y$ and $X$ attribute values in $R$. If the table $R$ has a big size, full retrieval of its attribute values may take long time even if feasible. This observation motivates us to propose a scheme that can save equivalence probability estimation from the computation of $f(Y, X)$. The proposed scheme is called estimation by reference, which only involves the attribute values at $X$ correlated with $y_{1}$ and $y_{2}$ in $R, X\left[y_{1}\right]$ and $X\left[y_{2}\right]$.

Estimation by Reference Given two random tuples $r_{1 i}, r_{1 j} \in R\left[y_{1}\right]$, the probability of their attribute values at $Y$ being equivalent can be estimated by Eq. (5) as follows:
$P\left(y_{1 i} \simeq y_{1 j}\right) \approx f(Y, X) \cdot P\left(x_{1 i}=x_{1 j}\right)$
where $y_{1 i}$ and $y_{1 j}$ represent $r_{1 i}$ and $r_{1 j}$ 's attribute values at $Y$, and $x_{1 i}$ and $x_{1 j}$ represent their attribute values at $X$. Since $y_{1 i}=y_{1 j}=$ $y_{1}, p\left(y_{1 i}=y_{1 j}\right)=1$. Therefore, by combining Eqs. (5) and 10 , we can estimate the equivalence probability between $y_{1}$ and $y_{2}$ by
$P\left(y_{1} \simeq y_{2}\right) \approx \frac{P\left(x_{1}=x_{2}\right)}{P\left(x_{1 i}=x_{1 j}\right)}$
in which $P\left(x_{1}=x_{2}\right)$ represents the probability of a random value in $X\left[y_{1}\right]$ being identical to a random value in $X\left[y_{2}\right]$, and $P\left(x_{1 i}=\right.$ $x_{1 j}$ ) represents the probability of two random values in $X\left[y_{1}\right]$ being identical.

In Eq. (11), the denominator serves as the reference to determine the equivalence between $y_{1}$ and $y_{2}$. The closer the value of $P\left(x_{1}=x_{2}\right)$ is to the value of $P\left(x_{1 i}=x_{1 j}\right)$, the more probably $y_{1}$ and $y_{2}$ are equivalent. Suppose that in the running example, $y_{1}=$ "VLDB J" and $y_{2}=$ "Journal on Very Large Data Bases". Then, $P\left(x_{1}=x_{2}\right)$ measures the title similarity between the papers published at "VLDB J" and those published at "Journal on Very Large Data Bases", and $P\left(x_{1 i}=x_{1 j}\right)$ captures the title similarity among the papers published at "VLDB J". If these two title similarity measurements are very close in value, it can be expected that $y_{1}$ and $y_{2}$ refer to the same journal entity with a high probability.

We observe that both $R\left[y_{1}\right]$ and $R\left[y_{2}\right]$ can serve as the reference. In a practical implementation, we suggest to choose the one with the highest value of $P\left(x_{1 i}=x_{1 j}\right)$. We can represent the equivalence probability of $y_{1}$ and $y_{2}$ by
$P\left(y_{1} \simeq y_{2}\right) \approx \frac{P\left(x_{1}=x_{2}\right)}{\max _{a=1,2} P\left(x_{a i}=x_{a j}\right)}$
The probabilistic metric obtained by Eq. (12) is called as Value Correlation Analysis or $\operatorname{VCA}\left(y_{1}, y_{2}\right)$, which can be taken as the ratio of matching probability across clusters and the maximal counterpart within clusters.

### 3.1.2. An extension of basic model

The basic model as presented in Eq. (5) has limited applicability because it only considers identical attribute values. Consider


Fig. 1. An illustrative examples for $V C A$.
the running example shown in Table 1. To determine the equivalence between two non-identical journal names, we compare their corresponding paper titles. In case that their papers do not share common titles, the basic model for $\operatorname{VCA}\left(y_{1}, y_{2}\right)$ would approximate to be 0 . However, it can be observed that if two papers are published in the same journal, they may be in the same research area with a high probability; accordingly, their titles may be also similar. Therefore, it is important for VCA to capture the correlation between journal and title as shown in Table 1. To this end, we extend the basic model by incorporating string similarity measurement into the estimation of VCA.

The extended model considers not only identical values, but also similar values in computing the VCA in Eq. (5). Previous work on attribute value matching assumes that the probability of two attribute values being equivalent equals to their string similarity. Under such assumption, $P\left(x_{1}=x_{2}\right)$ in Eq. (5) can be substituted by $P\left(x_{1} \approx x_{2}\right)$, which represents the average of string similarity of pairs across $X\left[y_{1}\right]$ and $X\left[y_{2}\right]$, and can be represented by
$P\left(x_{1 i} \approx x_{2 j}\right)=\frac{\sum_{i, j} \operatorname{sim}\left(x_{1 i}, x_{2 j}\right)}{\left|X\left[y_{1}\right]\right| \cdot\left|X\left[y_{2}\right]\right|}$
in which $x_{1 i} \in X\left[y_{1}\right], x_{2 j} \in X\left[y_{2}\right]$ and $\operatorname{sim}\left(x_{1 i}, x_{2 j}\right)$ specifies the string similarity between $x_{1 i}$ and $x_{2 j}$. Note that in Eq. (13), the numerator exhaustively aggregates the similarity of every value pair across $X\left[y_{1}\right]$ and $X\left[y_{2}\right]$, and the denominator represents the cardinality of Cartesian product $X\left[y_{1}\right] \times X\left[y_{2}\right]$.

Similarly, $P\left(x_{a i} \approx x_{a j}\right)$ can be expressed by
$P\left(x_{a i} \approx x_{a j}\right)=\frac{\sum_{i, j} \operatorname{sim}\left(x_{a i}, x_{a j}\right)}{\left|X\left[y_{a}\right]\right| \cdot\left(\left|X\left[y_{a}\right]\right|-1\right) / 2}$
where $x_{a i}$ and $x_{a j}(i \neq j)$ represent two random values from $X\left[y_{a}\right]$, and the denominator denotes the cardinality of value pairs.

By extending Eq. (12) with Eq. (13) and Eq. (14), we obtain Eq. (15).
$V C A\left(y_{1} \simeq y_{2}\right)=\frac{P\left(x_{1 i} \approx x_{2 j}\right)}{\max _{a=1,2}\left(P\left(x_{a i} \approx x_{a j}\right)\right)}$
Analogously, $\operatorname{VCA}\left(y_{1}, y_{2}\right)$ in extended model, can be regarded as the ratio of the average similarity across clusters and the maximal counterpart within clusters.

Example 1. As shown in Fig. 1, assume that there are many research papers included in publication venue "VLDB" (denoted
by $y_{1}$ ), and "Very Large Database" (denoted by $y_{2}$ ) respectively. Since $y_{1}$ and $y_{2}$ refer to the same entity of Journal, $X\left[y_{1}\right], X\left[y_{2}\right]$, $X\left[y_{1} \cup y_{2}\right]$ were deemed to be drawn from the same underlying distribution. Should sufficient samples be available, VCA for pairs that are equivalent, e.g., $\operatorname{VCA}\left(y_{1}, y_{2}\right)$, tends to approximate to 1 . while $V C A$ for pairs that are not equivalent, e.g., $\operatorname{VCA}\left(y_{1}, y_{3}\right)$ or $\operatorname{VCA}\left(y_{3}\right.$, $y_{4}$ ), are far less than 1 .

On the complexity of VCA, we have following lemmas:
Lemma 1. VCA $\left(y_{1}, y_{2}\right)$ takes the complexity of $O\left(\left(N_{1}+N_{2}\right)^{2}\right)$, where $N_{1}$ and $N_{2}$ denote the cardinalities of $X\left[y_{1}\right]$ and $X\left[y_{2}\right]$ respectively.

Lemma 2. $\operatorname{VCA}\left(y_{i}, y_{j}\right)$ over all value pairs takes the complexity of $O\left(N^{2}\right)$, where $N$ denotes the total number of tuples and pair $\left\langle y_{i}\right.$, $\left.y_{j}\right\rangle \in\langle\mathbf{Y} \times \mathbf{Y}\rangle$.

### 3.1.3. Reasoning by multiple attributes

To reason about the equivalence between attribute values by multiple correlated attributes, a straightforward solution is to combine multiple attribute values into a long field and then treat the resulting string as a single attribute in correlation analysis. However, this approach does not differentiate the uneven importance of different attributes. Using a single string similarity metric, it cannot incorporate specific domain knowledge on individual attributes into the reasoning process either.

To address the limitation of the straightforward solution, we model the problem of probabilistic equivalence reasoning as a classification problem. It classifies each value pair, $y_{i}$ and $y_{j}$, at a target attribute $Y$ into one of two labels, equivalent and inequivalent. Other attributes in $R$ (except $Y$ ) are considered as the feature sets of the objects (value pairs). Probabilistic reasoning by an individual attribute works as a classifier, which computes for its corresponding feature. Probabilistic reasoning by multiple attributes then corresponds to the problem of building an ensemble of classifiers and combining their classification results.

According to the ensemble theory [15], the performance of an ensemble depends on the performance of its component classifiers and their performance diversity. Therefore, the process of attribute selection for probabilistic reasoning consists of two steps. The first step of attribute filtering chooses the candidate attributes whose corresponding classifiers can achieve good performance. The following step of combination generation selects a combination of attributes from the candidate set with the aim to boost the performance diversity among the chosen attributes.

Attribute Filtering Informative attributes should have two properties:

1. high similarities for pairs drawn from the same clusters;
2. low similarities for pairs across clusters.

Informative attributes tend to provide strong evidence for matching attribute values. We represent such informativeness by conditional correlation factor (ccf).

We first define the conditional correlation factor between attribute $X$ and $Y_{1}$ by
$\operatorname{ccf}\left(X, y_{1}\right)=\frac{P\left(x_{1 i} \approx x_{1 j}\right)}{P\left(x_{i} \approx x_{j}\right)}$
where $x_{1 i}, x_{1 j} \in X\left[y_{1}\right]$ and $x_{i}, x_{j} \in X[R]$.
Given a target attribute $Y$ and a candidate attribute $X$, we measure their conditional correlation by a weighted sum of $\operatorname{ccf}\left(X, y_{a}\right)$ as follows:
$c c f(X, Y)=\sum_{y_{a} \in \mathbf{Y}}\left(w_{a} \cdot c c f\left(X, y_{a}\right)\right)$
in which $y_{a}$ denotes a distinct value in domain $\mathbf{Y}, w_{a}$ denotes its weight, which is proportional to the occurrence frequency of $y_{a}$ in
$Y[R]$. Intuitively, high value of $c c f$ implies that high similarities for pairs drawn from the same clusters and low similarities for pairs drawn across clusters.

The process of attribute selection sets a threshold $\theta_{c}$ (e.g., $\theta_{c}=$ 5 ) on the value of measured conditional correlation. An attribute $X$ will be included into the candidate set if and only if its conditional correlation with regard to $Y$ is no smaller than $\theta_{c}, \operatorname{ccf}(X, Y) \geq \theta_{c}$. Intuitively, if the $Y$ attribute values are independent of their corresponding $X$ values, the value of $c c f(X, Y)$ tend to achieve to 1 .

Combination Generation
We first describe how to measure performance diversity among candidate attributes, and then present an algorithm to generate the attribute combination.

According to Eq. (5), given a candidate attribute, $X_{1}$, the equivalence probability of a pair of target $Y$ attribute values, $\left(y_{i}, y_{j}\right)$, as reasoned by $X_{1}$, can be represented by
$P\left(y_{i} \simeq y_{j}\right)=f\left(Y, X_{1}\right) \cdot P\left(x_{1 i}=x_{1 j}\right)$
in which $x_{1 i} \in X_{1}\left[y_{i}\right], x_{1 j} \in X_{1}\left[y_{j}\right]$ and $P\left(x_{1 i}=x_{1 j}\right)$ represents the equivalence probability between a random value in $X_{1}\left[y_{i}\right]$ and a random value in $X_{1}\left[y_{j}\right]$. Therefore, given two candidate attributes, $X_{1}$ and $X_{2}$, their performance difference on reasoning about the equivalence between $y_{i}$ and $y_{j}, D_{\left(y_{i}, y_{j}\right)}\left(X_{1}, X_{2}\right)$ can be represented by
$\left|f\left(Y, X_{1}\right) \cdot P\left(x_{1 i}=x_{1 j}\right)-f\left(Y, X_{2}\right) \cdot P\left(x_{2 i}=x_{2 j}\right)\right|$
Additionally, denoting $P\left(y_{i}=y_{j}\right)$ and $P\left(x_{i}=x_{j}\right)$ by $P(Y(R))$ and $P(X(R))$ respectively, we have
$f\left(Y, X_{1}\right)=\frac{P(Y[R])}{P\left(X_{1}[R]\right)}$
$f\left(Y, X_{2}\right)=\frac{P(Y[R])}{P\left(X_{2}[R]\right)}$
$f\left(X_{1}, X_{2}\right)=\frac{P\left(X_{1}[R]\right)}{P\left(X_{2}[R]\right)}$
Therefore, we have
$f\left(Y, X_{2}\right)=f\left(Y, X_{1}\right) \cdot f\left(X_{1}, X_{2}\right)$
Normalized by the estimation result by $X_{1}$, the performance difference between $X_{1}$ and $X_{2}$ can be rewritten as
$D_{\left(y_{i}, y_{j}\right)}\left(X_{1}, X_{2}\right)=\left|1-f\left(X_{1}, X_{2}\right) \cdot \frac{P\left(x_{2 i}=x_{2 j}\right)}{P\left(x_{1 i}=x_{1 j}\right)}\right|$
Denoting the correlation between $X_{1}\left(y_{i}\right)$ and $X_{2}\left(y_{j}\right)$ by
$f\left(X_{1}\left[y_{i}\right], X_{2}\left[y_{j}\right]\right)=\frac{P\left(x_{1 i}=x_{1 j}\right)}{P\left(x_{2 i}=x_{2 j}\right)}$
we can therefore rewrite Eq. (24) as
$D_{\left(y_{i}, y_{j}\right)}\left(X_{1}, X_{2}\right)=\left|1-\frac{f\left(X_{1}, X_{2}\right)}{f\left(X_{1}\left[y_{i}\right], X_{2}\left[y_{j}\right]\right)}\right|$
Since probabilistic reasoning aims to identify equivalent attribute values, the classifier ensemble should be designed to boost the performance diversity of the classifiers on these value pairs. Therefore, we estimate the performance diversity between $X_{1}$ and $X_{2}$ by considering all the identical value pairs at $Y,\left(y_{i}, y_{i}\right)$. Denoting $D_{\left(y_{i}, y_{i}\right)}\left(X_{1}, X_{2}\right)$ by $D_{y_{i}}\left(X_{1}, X_{2}\right)$, we have
$D_{y_{i}}\left(X_{1}, X_{2}\right)=\left|1-\frac{f\left(X_{1}, X_{2}\right)}{f\left(X_{1}\left[y_{i}\right], X_{2}\left[y_{i}\right]\right)}\right|$
Therefore, we can estimate the performance diversity between $X_{1}$ and $X_{2}$ on all the identical $Y$ value pairs by
$D_{Y}\left(X_{1}, X_{2}\right) \approx \sum_{y_{i} \in \mathbf{Y}} w_{i} \cdot D_{y_{i}}\left(X_{1}, X_{2}\right)$
in which $y_{i}$ represents a distinct value in $\mathbf{Y}$, and $w_{i}$ represents its weight, whose value is proportional to the occurrence frequency of $y_{i}$ in $\mathbf{Y}$.

The algorithm for generating attribute combination is sketched in Algorithm 1 . It iteratively selects the attribute $X_{j}$ with the high-

```
Algorithm 1: Algorithm for Generating Attribute Combination.
    // \(S_{C}\) denotes the candidate attribute set;
    // \(S_{F}\) denotes the set of chosen attributes;
    // Y denotes the target attribute;
    \(S_{F}=\emptyset\);
    while \(\left(S_{C} \neq \emptyset\right)\) do
        Select the attribute \(X_{j}\) in \(S_{C}\) with the highest conditional
        correlation factor with regard to \(Y, c c f\left(X_{j}, Y\right)\);
        if \(S_{F}=\emptyset\) then
            Insert \(X_{j}\) into \(S_{F}\);
        else
            if (there does not exist \(X_{i} \in S_{F}\) s.t. \(\left.D_{Y}\left(X_{i}, X_{j}\right)<\theta_{d}\right)\) then
                    Insert \(X_{j}\) into \(S_{F}\);
        Remove \(X_{j}\) from \(S_{C}\);
```

est conditional correlation factor in the candidate attribute set $S_{C}$. If there exists an attribute $X_{i}$ in the result attribute set, $S_{F}$, such that the performance diversity between $X_{i}$ and $X_{j}$ with regard to $Y$ as estimated by Eq. (28) is below a threshold $\theta_{d}$ (e.g., 0.3 ), $D_{Y}\left(X_{i}\right.$, $\left.X_{j}\right)<\theta_{d}$, then $X_{j}$ is filtered out; otherwise, $X_{j}$ is inserted into $S_{F}$. The algorithm stops when the candidate set $S_{C}$ becomes empty.

On the complexity of Algorithm 1, we have following lemma:
Lemma 3. Provided that performance diversities have been computed beforehand, Algorithm 1 runs in $O\left(k^{2}\right)$ time in worse case, in which $k$ denotes the number of attributes in $R$.

## Combination Rule

As in other applications of the ensemble theory, the probabilistic estimations computed by the classifiers for individual attributes are combined using the voting method. It first estimates the equivalence probability of $y_{1}$ and $y_{2}$ by each chosen attribute and then computes a weighted sum of the estimation results. The weight for an attribute $X_{i}$ is proportional to the $\log$ of the conditional correlation factor between $X_{i}$ and $Y$. Suppose that the chosen $k$ attributes are $\left\{X_{1}, \ldots, X_{k}\right\}$. The combined probability is estimated by
$\operatorname{VCA}\left(y_{1} \simeq y_{2}\right)=\sum_{1 \leq i \leq k} w_{i} \cdot V C A_{i}\left(y_{1} \simeq y_{2}\right)$
$V C A_{i}\left(y_{1} \simeq y_{2}\right) X_{i} w_{i} X_{i} 29 w_{i}$
$w_{i}=\frac{\ln \left(c c f\left(X_{i}, Y\right)\right.}{\sum_{1 \leq i \leq k} \ln \left(c c f\left(X_{i}, Y\right)\right)}$
$\operatorname{ccf}\left(X_{i}, Y\right) X_{i} \ln (\cdot)$

### 3.2. The unified framework

The unified framework consists of two analytical components: string similarity measurement and value correlation analysis. We first describe a theory of evidence, the Dempster-Shafer theory, in Section 3.2.1, and then present how to use it to combine the estimation results obtained at the two components in Section 3.2.2.

### 3.2.1. The Dempster-Shafer theory

The Dempster-Shafer ( $D-S$ ) theory $[16,17]$, which is a generalization of the Bayesian theory of subjective probability, combines evidences from different sources and arrives at a degree of belief that takes into account all the available evidence.

Formally, let $Z$ be the universal set representing all possible states of a system under consideration. By a function of Basic Belief Assignment (BBA), the D-S theory assigns a belief mass to each element of the power set. The mass of an element $E_{i}, m\left(E_{i}\right)$, expresses the proportion of all relevant and available evidence that supports the claim that the actual state belongs to $E_{i}$ but to no particular subset of $E_{i}$. The masses of elements satisfy
$\sum_{E_{i} \in 2^{z}} m\left(E_{i}\right)=1$,
and
$m(\emptyset)=0$.
If only singleton propositions are assigned belief masses, a BBA function reduces to a classical probability function.

The kernel of $D-S$ theory is Dempster's rule, which is rooted in probability theory and constitutes a conjunctive probabilistic inference process. It generalises Bayes' rule and was indeed promoted as the sole evidence combination rule to combine evidence in the $D-S$ framework originally. The Dempster's rule assumes that evidence is fully reliable: a proposition will not be supported at all if it is ruled out by an evidence. It adopts the orthogonal sum operation to combine evidence, which is rooted in calculating the joint probability of independent events. With two pieces of independent evidence represented by two BBAs $m_{1}$ and $m_{2}$ respectively, the joint mass of a proposition $E$ is calculated in the following manner:
$m_{1,2}(E)=\frac{\sum_{E_{i} \cap E_{j}=E \neq \emptyset} m_{1}\left(E_{i}\right) \cdot m_{2}\left(E_{j}\right)}{1-\sum_{E_{i} \cap E_{j}=\emptyset} m_{1}\left(E_{i}\right) \cdot m_{2}\left(E_{j}\right)}$,
and
$m_{1,2}(\emptyset)=0$.
in which $\sum_{E_{i} \cap E_{j}=\emptyset} m_{1}\left(E_{i}\right) \cdot m_{2}\left(E_{j}\right)$ measures the amount of conflict between the two mass sets.

### 3.2.2. Evidential reasoning

The unified framework assumes that the two pieces of evidences for attribute value equivalence, based on string similarity measurement and value correlation analysis respectively, are independent. It therefore uses the Dempster's rule of combination in evidential reasoning.

Suppose that the equivalence probability of two attribute values $y_{1}$ and $y_{2}$ estimated by string similarity measurement is denoted as $P_{s}\left(y_{1} \simeq y_{2}\right)$; similarly, the probability estimated by value correlation analysis is denoted as $P_{c}\left(y_{1} \simeq y_{2}\right)$. The BBA function of string similarity measurement assigns probability masses to singleton propositions in the following manner:
$m_{s}\left(y_{1} \simeq y_{2}\right)=P_{s}\left(y_{1} \simeq y_{2}\right)$,
and
$m_{s}\left(y_{1}!\simeq y_{2}\right)=1-P_{s}\left(y_{1} \simeq y_{2}\right)$.
Similarly, the BBA function of value correlation analysis assigns probability masses to singleton propositions in the following manner:
$m_{c}\left(y_{1} \simeq y_{2}\right)=P_{c}\left(y_{1} \simeq y_{2}\right)$,
and
$m_{c}\left(y_{1}!\simeq y_{2}\right)=1-P_{c}\left(y_{1} \simeq y_{2}\right)$.
Then, according to the Dempster's rule, the combined probability of $y_{1}$ and $y_{2}$ being equivalent is computed by
$p_{s, c}\left(y_{1} \simeq y_{2}\right)=\frac{p_{s} \cdot p_{c}}{1-p_{s} \cdot\left(1-p_{c}\right)-\left(1-p_{s}\right) \cdot p_{c}}$
in which $p_{s}$ denotes $p_{s}\left(y_{1} \simeq y_{2}\right)$ and $p_{c}$ denotes $p_{c}\left(y_{1} \simeq y_{2}\right)$
For instance, suppose that the equivalence probabilities estimated by similarity measurement and correlation analysis are 0.8 and 0.6 respectively. Then, the combined probability is computed by $\frac{0.8 \cdot 0.6}{1-0.8 \cdot 0.4-0.2 \cdot 0.6}=0.857$.

According to the above formula, the unified estimation would be 0 if any of the individual estimations is computed to be 0 . To address this counter-intuitive issue, we introduce evidence weights and translate them into simple support functions. We use the Shafer's discounting method [16] to combine two weighted evidence. Suppose $p\left(E_{i}\right)$ is the degree of belief to which a piece of evidence points to a proposition $E_{i}$. Let $w$ be a factor that is used to discount $p\left(E_{i}\right)$, where $w(0 \leq w \leq 1)$ is interpreted as evidence weight. The Shafer's discounting method defines the BBA function for an evidence as follows:
$m\left(E_{i}\right)= \begin{cases}w \cdot p\left(E_{i}\right) & E_{i} \subset Z, E_{i} \neq \emptyset \\ 0 & E_{i}=\emptyset \\ 1-w & E_{i}=Z\end{cases}$
What Eq. (32) means is that the degree of support for a proposition is proportional to both the weight of the evidence and the belief degree to which the evidence points to the proposition. It allocates the residual support left by the evidence due to its weight, as measured by $(1-w)$, to $Z$. We have noticed that there are other methods [17] for weighted evidence combination. The difference is on how to handle the residual support, as measured by $1-w$. We propose to use the Shafer's discounting method because assigning the residual support to $Z$ is most reasonable in this setting.

Generally, suppose that the evidence weights of similarity measurement and correlation analysis are $w_{s}$ and $w_{c}$ respectively. Then, we have
$m_{s}\left(y_{1} \simeq y_{2}\right)=w_{s} \cdot p_{s}$,
$m_{s}\left(y_{1}!\simeq y_{2}\right)=w_{s} \cdot\left(1-p_{s}\right)$,
and
$m_{s}(Z)=1-w_{s}$.
Similarly, we also have
$m_{c}\left(y_{1} \simeq y_{2}\right)=w_{c} \cdot p_{c}$,
$m_{c}\left(y_{1}!\simeq y_{2}\right)=w_{c} \cdot\left(1-p_{c}\right)$,
and
$m_{s}(Z)=1-w_{c}$.
Therefore, the unified equivalence probability can be computed by

$$
\begin{align*}
& p_{s, c}\left(y_{1} \simeq y_{2}\right) \\
& =\quad \frac{w_{s} \cdot p_{s} \cdot w_{c} \cdot p_{c}+w_{s} \cdot p_{s} \cdot\left(1-w_{c}\right)+w_{c} \cdot p_{c} \cdot\left(1-w_{s}\right)}{1-w_{s} \cdot p_{s} \cdot w_{c} \cdot\left(1-p_{c}\right)-w_{c} \cdot p_{c} \cdot w_{s} \cdot\left(1-p_{s}\right)} \tag{33}
\end{align*}
$$

Consider again the case that the equivalence probabilities estimated by similarity measurement and correlation analysis are 0.8 and 0.6 respectively; but both evidence weights are 0.5 . Then, we have
$m_{s}\left(y_{1} \simeq y_{2}\right)=0.4$,
$m_{s}\left(y_{1}!\simeq y_{2}\right)=0.1$,
and
$m_{s}(Z)=0.5$.
We also have
$m_{c}\left(y_{1} \simeq y_{2}\right)=0.3$,
$m_{c}\left(y_{1}!\simeq y_{2}\right)=0.2$,
and
$m_{c}(Z)=0.5$.
The combined probability is therefore computed by
$m_{s, c}\left(y_{1} \simeq y_{2}\right)=\frac{0.4 \cdot 0.3+0.4 \cdot 0.5+0.3 \cdot 0.5}{1-0.4 \cdot 0.2-0.3 \cdot 0.1}=0.528$
Our experimental evaluation in Section 4.3 shows that evidence weighting can affect the performance of the unified framework and it performs best when the two weights are set to be equal. Therefore, we suggest to set the weights of string similarity measurement and value correlation analysis to be equal in practical implementation. It is worthy to point out that the unified framework is reduced into string similarity measurement or value correlation analysis if the weight of the other component is set to be 0 .

## 4. Experimental study

In this section, we empirically evaluate the performance of the unified framework on real datasets. The details of these test datasets are described as follows:

1. DBLP. The DBLP dataset ${ }^{1}$ contains the information on the papers published in the research venues in various areas of computer science. The original dataset is clean, containing no dirty data. For experimental purpose, we introduce dirty data by manipulating the attribute values at journal, which specifies the venue a paper is published in. We transform the journal values by performing a series of edit operations, e.g. insertion, deletion and substitution, as defined in the Edit distance[18]. We specify the numbers of edit operations executed on journal values by a normal distribution, $\mathcal{N}\left(\mu, \sigma^{2}\right)$, which we refer to as the manipulation function in the rest of this section.
2. HOTEL. The dataset contains the information of more than 138,000 hotels around the world. Similar to the case of DBLP, we manually generate dirty data by manipulating the attribute values at city and aircode, which specify the city a hotel is located at and the code of its nearest airport respectively. As on DBLP, we transform the values by performing a series of edit operations as defined in the Edit distance. The numbers of executed edit operations on the city values are also specified by a normal distribution, $\mathcal{N}\left(\mu, \sigma^{2}\right)$. The aircode values have only three characters. Therefore, we randomly replace 1 or 2 characters at the chosen values.
3. DBLP+CiteSeer (DLCS). Similar to DBLP, the CiteSeer dataset ${ }^{2}$ also records the published research papers in computer science. An author and a publication venue may have different representations in DBLP and CiteSeer. Our experiments aim to match the attribute values of journal and author between them. DBLP and CiteSeer contain around 960,000 and 45,000 tuples respectively. To facilitate repeated experimentation, we randomly choose $20 \%$ tuples in DBLP and match them with all the tuples in CiteSeer.
4. CORA. The CORA dataset ${ }^{3}$ also contains the published research papers in computer science. Unlike the DBLP and CiteSeer datasets, it is pretty dirty by itself. In CORA, two records may refer to the same paper entity but their publication venues have different representations. The dataset contains totally 1295

[^1]

Fig. 2. Precision-cutoff evaluation on DBLP.
records and 474 attribute values at venue, which specifies publication venue. Our experiments aim to match the attribute values at venue. Since the venue values in CORA carry time information, two venues are considered to be different if they correspond to the same conference/journal series but have distinct publication time (e.g. "the 31th IEEE International Conference on Data Engineering" and "the 32nd IEEE International Conference on Data Engineering").

Our experiments mainly compare the performance of four approaches, string similarity measurement (SSM), value correlation analysis (VCA) and the unified framework (TUF). Additionally, we compare TUF to the approach of collective entity resolution (CER) proposed in [19], which takes advantage of co-occurring references to reason about entity equivalence jointly rather than independently. Note that the effectiveness of the CER approach depends on the presence of value co-occurrence in a record. Compared with TUF, CER has more limited applicability in practice. For instance, CER can only work on the author names of the bibliographic datasets, but not on their journal attribute values; CER can't work on the HOTEL dataset either. In comparison, TUF can work on both the author and journal attributes of the bibliographic datasets, and it can also work on the HOTEL dataset.

### 4.1. Comparative evaluation

In this subsection, we first compare the performance of SSM, VCA and TUF (Section 4.1.1 on the DBLP dataset, Section 4.1.2 on HOTEL and Section 4.1.3 on DLCS) and then compare TUF to CER (Section 4.1.4). The unified framework sets both weights of SSM and VCA to the default value of 0.5 . On the DBLP dataset, VCA reasons about journal by title and author. On HOTEL, VCA reasons about city by address, and aircode by address and city. On DLCS, VCA reasons about journal by title and author, and author by title and journal. On CORA, VCA reasons about venue by title and author. Reasoning over multiple attributes are presented in Section 3.1.3.

### 4.1.1. DBLP

To evaluate the comparative performance in the circumstances where SSM has varying accuracies, we set the value of standard deviation ( $\sigma$ ) of the manipulation function to 2 but vary its value of mean ( $\mu$ ) from 3 to 7 . Note that as the value of $\mu$ increases, the equivalent attribute values would appear less similar. As a result, the performance of SSM would deteriorate.

The precisions measured over top-K (cutoff) are presented in Fig. 2. We present the precision results at the cutoff levels up to 100, where TUF achieves the maximal recalls. In the case of $\mu=3$, it can be observed that even though SSM performs better than VCA, TUF performs clearly better than SSM, and the performance gain ranges from $8 \%$ to $30 \%$. Close scrutiny reveals that VCA tends to match the journals in the same research area, whose papers have similar titles and share some common authors. It effectively filters


Fig. 3. Precision-cutoff evaluation on HOTEL.
out the journal names that appear similar but are actually in different research areas from its top ranked list.

It can also be observed that as the value of $\mu$ increases, the performance of SSM deteriorates as expected while the performance of VCA remains stable. As a result, the performance improvement achieved by TUF over SSM becomes more considerable. As the value of $\mu$ increases, SSM becomes less reliable in determining value equivalence. The performance of VCA is instead independent of the representations of values on journal. Since TUF leverages both SSM and VCA estimation results, its performance improvement over SSM increases as the performance of SSM deteriorates.

### 4.1.2. HOTEL

On the city values, the mean and standard deviation of the manipulation function are set to be 3 and 2 respectively, $\mu=3$ and $\sigma=2$. The aircode values have only three characters. Therefore, we randomly replace 1 or 2 characters at the chosen values.

The comparative precision-cutoff evaluation results are presented in Fig. 3. We report the precision results up to top-1000. After the cutoff level of 1000, every approach generates few ( $\leq 5$ ) correct matches.

It can be observed that on city, even though neither SSM nor VCA achieves the precision above $50 \%$, TUF achieves a precision above $90 \%$ at the cutoff level of 100 . On aircode, it can be observed that SSM performs very poorly. With only three characters, many aircode value pairs have the same string similarity but most of them actually represent different airports. There are a lot of false positive matches in the top-k results. As a result, the performance of SSM remains low (around $10 \%$ ) at various cutoff levels. In comparison, by analyzing the correlated city and address values, VCA achieves much higher matching accuracy. It can also be observed that TUF's performance gains over SSM range from $5 \%$ to $50 \%$ over city, and from $11 \%$ to $70 \%$ over aircode respectively.

### 4.1.3. DLCS \& CORA

On the DLCS and CORA datasets, we evaluate the precisioncutoff performance of different approaches. On one hand, manually


Fig. 4. Precision-cutoff evaluation on DLCS.


Fig. 5. Precision-cutoff evaluation on CORA.
checking the true recall values on these datasets is a daunting task. On the other hand, if the cutoff level is set to be large enough, the percentage of matching pairs among those ranked after the cutoff level could be expected to be very small; therefore, further increasing the cutoff level could only have negligible effect on the comparative performance of different approaches. In our experiments, the cutoff levels of DLCS and CORA are set to be 400 and 200 respectively. After the cutoff levels, the percentage of matching pairs becomes small enough such that the comparative performance of different approaches stabilize. The experimental results on DLCS are shown in Fig. 4 (a) and (b). Similar to what were observed on DBLP and HOTEL, TUF consistently outperforms SSM on matching precision. On journal, SSM performs well at the cutoff levels of 40 and 80 . It achieves the precisions of $97.5 \%$ and $87.5 \%$ respectively. At these two cutoff levels, TUF achieves slightly better performance than SSM. After that, TUF outperforms SSM by considerable margins: at the cutoff levels between 160 and 400, TUF achieves around $20 \%$ improvement over SSM.

On author, SSM does not perform as well as on journal. In DLCS, there are many author names that appear similar but actually represent different researchers. As a result, SSM returns many false positive matches in its top-k results. It can be observed that even though VCA performs worse than SSM, TUF performs much better than SSM. After the cutoff level of 80, TUF achieves at least $30 \%$ improvement over SSM on precision.

The experimental results on CORA are presented in Fig. 5. Since many values on venue values contain the time token either explicitly or implicitly, e.g. "ICDE,1998", "31st ICDE". Values are taken as equivalents only if they are matching on both of publication venue and time token (either explicit or implicit) in our experiment. It can be observed that SSM performs poorly. Ranking by SSM results in many false positive matchings in the top-k pairs. In


Fig. 6. TUF vs CER on Citeseer.
contrast, VCA performs well on CORA, achieving precision between $70 \%$ and $80 \%$ at various cutoff levels. By leveraging both analytical results of SSM and VCA, TUF achieves the best performance among them, and the performance gain of TUF over SSM ranges from $15 \%$ to $45 \%$.

### 4.1.4. TUF vs CER

Since the approach of CER can only work on the attribute values with the co-occurrence relationship, we compare the performance of TUF and CER on the author attribute of the single-source Citeseer dataset. In Citeseer, very few author names are ambiguous. Therefore, we ignore name ambiguity and suppose that identical author names refer to the same person. We have implemented the CER approach as presented in [19]. In the experiment, both CER and TUF used the hybrid metric, which augments a primary TF-IDF weighting scheme for matching token sets with a secondary JaroWinkler scheme for matching tokens, to measure string similarity. We have actually tested different secondary metrics for CER, and observed that as reported in [19], CER performs best with the secondary metric of Jaro-Winkler. As in Section 4.1.3, we measure the performance by the precisions at various cutoff levels. The comparative results are presented in Fig. 6. It can be observed that due to limited growth space, performance gain ( $2 \%$ ) seems small, however TUF performs consistently better than CER.

### 4.2. Attribute combination

This subsection evaluates the effectiveness of attribute selection and estimation combination in value correlation analysis, as presented in Section 3.1.3. On all the three datasets, the threshold of conditional correlation factor $\left(\theta_{c}\right)$ for candidate filtering is set to be 5 ; the threshold of the performance diversity between two candidate attributes $\left(\theta_{d}\right)$ for combination generation is set to be 0.3 . The conditional correlation factor of chosen attributes and their performance diversity with regard to a target attribute are detailed in Table 3.

The evaluation results on DBLP are presented in Fig. 7 (a) and (b). On DBLP, we set the value of $\mu$ to be 5 . The evaluation results for other values of $\mu$ are similar, thus omitted here. Value correlation analysis are executed on author, title and the combination of author and title respectively. In Fig. 7 (a), it can be observed that VCA on author performs better than VCA on title. Similarly, in Fig. 7 (b), it can be observed that TUF with VCA on author performs better than TUF with VCA on title. We also observe that VCA on the attribute combination clearly outperforms VCA on single attributes. The comparative performance of their corresponding TUF are similar. The evaluation results on

Table 3
The values of $\operatorname{ccf}(X, Y)$ on the Datasets.

| Dataset $(Y)$ | $\operatorname{ccf}\left(X_{1}, Y\right)$ | $\operatorname{ccf}\left(X_{2}, Y\right)$ | $D_{Y}\left(X_{1}, X_{2}\right)$ |
| :--- | :--- | :--- | :--- |
| DBLP(journal) $X_{1}=$ author $X_{2}=$ title | 24.70 | 14.84 | 0.48 |
| DLCS (journal) $X_{1}=$ author $X_{2}=$ title | 332.50 | 69.60 | 0.79 |
| HOTEL(aircode) $X_{1}=$ city $X_{2}=$ address | 440.14 | 93.15 | 0.81 |



Fig. 7. Precision-cutoff evaluation results of attribute combination on DBLP and HOTEL.

DLCS are also presented in Fig. 7 (c) and (d). They are similar to what were observed on DBLP.

On HOTEL, the value of $c c f($ city, aircode) is larger than the value of ccf(address,aircode). As a result, VCA on city performs better than VCA on address. VCA on the attribute combination outperforms both of them. TUF with VCA on the attribute combination also achieves the best performance.

Our experiments show that the higher conditional correlation factor a candidate attribute has, the better performance its corresponding VCA and TUF can achieve. Moreover, the performances of VCA and TUF are better on attribute combination than on any single attribute. They validate the effectiveness of our approach for value correlation analysis based on multiple attributes proposed in Section 3.1.3.

### 4.3. Evidence weighting

This subsection evaluates how evidence weighting can affect the performance of the unified framework. We present the evaluation results on matching journal values on the DBLP dataset. The results on other datasets are similar, thus omitted here due to


Fig. 8. Evidence weighting.


Fig. 9. Efficiency evaluation.
space limit. The $\mu$ and $\sigma$ values of the normal function are set to be 5 and 2 respectively. Our experiments showed that if the weights of SSM and VCA are set to be equal ( $0<w<1$ ), the performance of TUF remains stable regardless of the value of $w$. Therefore, we set the weight of one component to the default value of 0.5 and vary the weight of the other component from 0.1 and 0.5 .

The detailed results are presented in Fig. 8. From Fig. 8 (a), it can be observed that if the weight of SSM is set to be 0.5 , TUF achieves the best performance when the weight of VCA is also set to be 0.5 . TUF with any weight setting outperforms SSM and its performance consistently improves as the weight of VCA increases from 0.1 to 0.5 . The experimental results for the case of varying the SSM weight, as presented in Fig. 8 (b), are similar. TUF with both weights set to be the default value of 0.5 achieves the best performance.

### 4.4. Scalability evaluation

The algorithms were evaluated on DBLP. VCA reasons about journal by author. In the first case, we vary data size (measured by the total number of tuples) from 200,000 to 800,000 . In the second case, the number of tuples is set to 200, to better measure the scalability with the number of attributes, author attribute is duplicated multiple times and appended to the relation $R$, and the horizon-axis of Fig. 9 (b) denote number of duplicates.

The experimental results are presented in Fig. 9. From Fig. 9 (a), we observe that the consumed time of VCA increases nearly quadratically with data size (consistent with Lemma 2). From Fig. 9 (b), we observe that our approach scales linear with the number of attributes.

## 5. Related work

Attribute Values Matching is an important research problem in data quality community, and the state-of-the-art approaches for attribute value matching are mainly based on string similarity among attribute values. In general, string similarity metrics can be broadly classified into three categories[8]: character-based, tokenbased and hybrid. The character-based metrics treat an attribute value as a long string. The most typical ones include edit distance, affine gap cost [20] or the Jaro metric [10]. The token-based metrics, e.g., Jaccard Similarity, convert the strings to token sets and consider similarity metrics on these sets. The token-based metrics are usually more effective than the character-based ones in the circumstances where word order is unimportant. Similar to the token-based metrics, the hybrid metrics [21,22] convert the strings to token multisets. It also considers token similarity in measuring the similarity between these multisets. However, as mentioned in [8], different string similarity metrics usually have distinct suitable application domains.

Beside the string similarity measurement, we also take into consideration the evidential support from value correlation analysis, which is distinct from existing approaches: (I) Constraint-based approaches, e.g., [23-25], have been widely used in data cleaning. However, to match attribute values, the effectiveness of such deterministic approaches may heavily rely on the quality of constraints (either specified by experts or discovered by automatic algorithms) and the number of pairs of tuples captured by matching pattern. In comparison, value correlation analysis or VCA, takes a probabilistic approach and has much wider application scenario. Those matching candidates failing to be captured as violations, e.g., "Journal on Very Large Databases" vs "VLDB J" in Table 1, can still be further resolved by VCA. (II) Bayesian theory [26] is also widely used in pattern recognition and statistical inference. A significant distinction lies in that the labels are predefined and distinct in Bayesian classification, whereas in the task of attribute value matching, the equivalence between labels need to be reasoned about. Since Bayesian-based approaches can not be used directly for the task of matching attribute values, we propose the novel concept of value correlation analysis, which is rooted in Bayesian theory and finally reduced to the ratio of matching probability across clusters (e.g., venues) and the maximal counterpart within clusters.

Additionally, we also list extensively some other closely related research work.

Duplicate Record Detection Duplicate record detection have been extensively studied in the literature [3,27]. Most effective and widely used approaches [28-30] were built on value matching on individual attribute fields. They solved the additional problem of how to merge the matching results on attribute fields. Obviously, our work on attribute value matching is complementary to them in that determining attribute value equivalence can effectively improve the accuracy of record matching.

For efficient duplicate detection, there also exist some reasonably recent work on blocking techniques [31-33], which separate records into blocks likely to contain matching pairs such that unnecessary pair-wise comparisons can be avoided. It can be observed that our work on attribute value equivalence reasoning can be used to improve the blocking accuracy.

Schema Matching Schema matching $[34,35]$ studies the problem of identifying columns that represent the same concept in two relational tables. Effective techniques usually take a hybrid approach by leveraging different criteria (e.g. linguistic matching, instance-based matching, structured-based matching, constraintbased matching and rule-based matching). The task of instancebased matching studied in schema matching treats all the value instances belonging to an attribute as a set and measures the set similarity by token-based metrics. Its matching techniques are sim-
ilar to the techniques of string similarity measurement used for value equivalence reasoning in that both of them reason based on contents. In contrast, our work in this paper proposes to reason about two attribute values' equivalence by analyzing their correlation with other attribute values. On the other hand, given a schema matching result, our work can effectively reduce equivalence ambiguity between attribute values and thus improve the accuracy of record linkage.

Entity Name Disambiguation There also exist work on entity name disambiguation in relational data [19] or on Web [36,37]. They assumed that identical names may refer to different realworld entities and focused on distinguishing these names.

In the circumstance that two identical values may refer to two different values, equivalence reasoning is very challenging because every appearance of value should be assumed to correspond to a distinct entity at the outset. An alternative solution is to first cluster distinct values into equivalent groups, which is the topic of this paper, and then distinguish the values in the same group. It is worthy to point out that our proposed approach can be similarly used to distinguish two identical values. In the case that two identical values refer to two distinct entities, it can be supposed that they are correlated with largely different other values. The application of the proposed approach to value disambiguation is however beyond the scope of this paper. Using Information from External Knowledge Base The Web presents a huge pool of useful knowledge, and we observe that various systems $[38,39]$ have been designed for information extraction on Web. Those extracted information can be used either by adding more columns or rows, to enrich the relational data, on which value correlation analysis is run.

## 6. Conclusion

This paper first presents a novel probabilistic approach to reason about attribute value equivalence in relational data by value correlation analysis. It then proposes a unified framework that can leverage both string similarity measurement and value correlation analysis. Finally, our extensive experiments on real datasets have validated the efficacy of the unified framework.

In this paper, we have focused on equivalence reasoning on string values. Theoretically, the effectiveness of value correlation analysis does not depend on the data type of attribute values. Equipped with appropriate similarity metrics, the unified framework can also be used for equivalence reasoning on numerical values. However, the existing methods for capturing similarity between numerical values are rather primitive. Therefore, the efficacy of our proposed approach on numerical values needs to be further investigated in future work.

In our default setting, only one attribute is taken as target attribute, on which value pairs will be matched. In the case that value equivalence should be considered on multiple attributes, they can be processed one by one. However, determining value equivalence on multiple attributes simultaneously is an interesting problem, which may deserve much more complex model or algorithm, thus can be taken as an interesting research point in future.

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